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# Dynamics of congestion transition triggered by multiple walkers on complex networks

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**Abstract.** The congestion transition triggered by multiple walkers walking along the shortest path on complex networks is numerically investigated. These networks are composed of nodes that have a finite capacity in analogy to the buffer memory of a computer. It is found that a transition from free-flow phase to congestion phase occurs at a critical walker density $f_c$, which varies for complex networks with different topological structures. The dynamic pictures of congestion for networks with different topological structures show that congestion on scale-free networks is a percolation process of congestion clusters, while the dynamics of congestion transition on non-scale-free networks is mainly a process of nucleation.

**PACS.** 89.75.-k Complex systems – 87.23.Ge Dynamics of social systems – 89.75.Fb Structures and organization in complex systems

## 1 Introduction

Complex networks are an essential part of modern society. A large number of natural and artificial systems, ranging from large communication system (the Internet, the WWW), transportation infrastructures (highway and airline routes), biological systems (gene and protein interaction networks) to social interaction networks [1–3], can be described by concepts of complex networks consisting of nodes connected by links. In the past few years it has been observed that a variety of real-world networks exhibit characteristic topologies which deviate from random networks [4–7].

Transport of matter or information is a fundamental function for many real networks such as vehicular flow in a network of highways and delivery of data packets between nodes with limited capacity on a communication network such as the Internet. The model of the traffic-flow on a regular lattice (square lattice, Cayley tree etc.) [8] has been constructed based on routing algorithms of computer networks and extended to networks with inhomogeneous structure such as scale-free networks [9–11]. A different model considered the transport process as a number of walkers (or particles) moving in the network through its links without the consideration of node capacity, which is equivalent to a single particle in the network [12]. Recently the interaction between walkers was introduced [13] by assuming that each node can be occupied at most by one walker and the walkers move randomly on the network, and some analytical expressions for dynamical quantities of interest were obtained. However, this model is too highly idealized because in real communication networks, information packets do not travel randomly, and the capacity of each node is neither homogeneous, nor limited to one packet [9].

## 2 Model

We model the transport on networks as a number of walkers walking on networks composed of nodes that have a finite capacity $c_i$ [9], which is defined as the maximal number of walkers (packets in communication networks) the $i$th node can accommodate at any time, in analogy to the buffer memory of a computer. Each walker selects a random destination among all nodes in the network and walks along the shortest path from the current position to the destination. The walker density $f$ is defined as the ratio of the number of walkers $n$ to the total capacity $C_{tot}$ of the network, i.e. $f = n/C_{tot}$, where $C_{tot} = \sum_i c_i$. The purpose of the present study is to see how the walker density affects the dynamics of flow and congestion on complex networks, and to draw the dynamic pictures of congestion for networks with different topological structures.

The simulations were carried out on scale-free networks, regular networks, small-world networks, and random networks. We appoint the network a total capacity and distribute them to each node on the network (we assumed that the total capacity is always multiple times of

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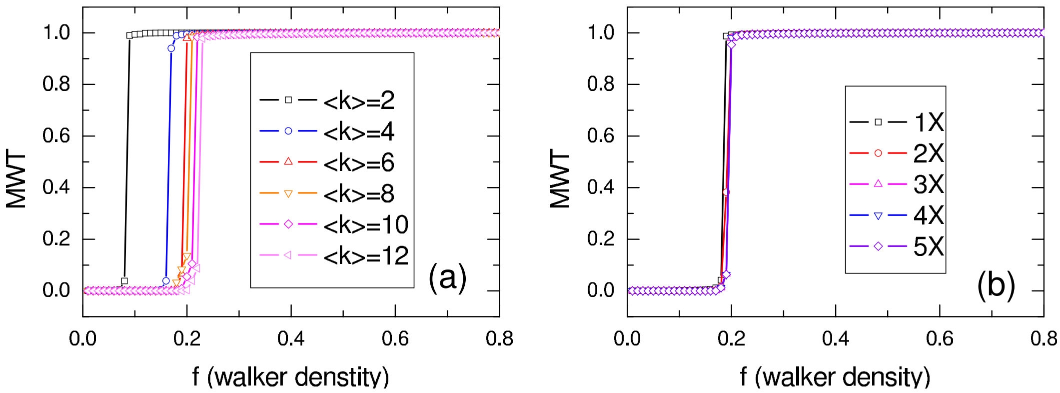


**Fig. 1.** (Color online) MWT as a function of walker density $f$ for BA scale-free networks with (a) different average degree $\langle k \rangle = 2 \sim 12$; (b) different total capacity $C_{tot} = (1 \sim 5)\sum_i k_i$. The number of time step is 10 000.

the total degree, such that $C_{tot} = l \sum_i k_i$, where $l$ is an integer and $k_i$ is the degree of node $i$). The capacity of node $i$ for the maximum number of walkers to reside is proportional to $\prod_i = k_i / \sum_j k_j$, which is also the probability for a walker to initially reside in node $i$. After an initial distribution of $n$ walkers on the network, each walker begins to randomly choose a node as its destination, then walks along one of the shortest paths from the current node to the destination. Each time step corresponds to one step along the shortest path the walker tries to walk. If the node that a walker tries to enter is full, the walker has to stop and reside in the current node at which it stays and wait to take another try till the next time step. Whereas if a node is unfilled and there are too many walkers approaching at the same time, then some are chosen randomly to enter the node and the others have to stop and stay in their corresponding current nodes. Once a walker has reached its destination, it chooses another destination at random and starts another journey. In our model, the walker density is kept constant during the walking process. To describe the degree of congestion for a network, we introduce a parameter, mean waiting time (MWT), which is defined as the average number of steps that a walker has to stay in its current node over the total number of simulation time steps $t_{tot}$, that is MWT $= \langle t_{waiting}/t_{tot} \rangle$.

## 3 Congestion on BA scale-free networks

A scale-free network is generated on the basis of the algorithm proposed by Barabási and Albert (BA model) [6]. Congestion on a BA scale-free network can be better understood if the simulations with different average degree $\langle k \rangle$ and total capacity $C_{tot}$ ($N = 1000$, $\langle k \rangle = 6$ and $C_{tot} = 3\sum_i k_i$ in the following simulations unless specified) are carried out. As shown in Figure 1, when the walker density is low, the MWT is very small, corresponding to a free flow phase. As the walker density on the network increases to a critical value $f_c$, MWT experiences a sudden jump, indicating a transition from a free-flow phase to a congestion phase of the system. The critical density $f_c$ can be regarded as the effective utility of the network capacity, which describes the maximal number of walkers a network can tolerate, or equivalently, how much redundancy of capacity we have to set aside in order to ensure a free flow on the network. It is found that the effective utility of network capacity on a scale-free network is very low without introducing new mechanisms to avoid congestion. For example, to guarantee the free flow on a scale-free network with $N = 1000$ and $\langle k \rangle = 6$, 80% of the total capacity has to be set aside as redundancy. To shift up the critical point $f_c$, one sees an increase of $f_c$ on scale-free networks when the average degree $\langle k \rangle$ increases (Fig. 1a). But $f_c$ is not sensitive to variations of the total capacity $C_{tot}$ (Fig. 1b), which means that we can improve the effective utility of network capacity by adding more links to the network. But as can be seen from Figure 1a, this method can only be effectively used on relatively sparse networks as $f_c$ begins to saturate when $\langle k \rangle \geq 6$, in analogy to the phenomenon of diminishing marginal utility in the field of economics.

Let's go deeper into what happens when the congestion occurs on complex networks. Figure 2a shows the number of congested nodes (the filled nodes) $N_{cong}$, caused by the initial distribution of walkers on a BA scale-free network with $N = 1000$ and $\langle k \rangle = 6$. It is found that even if the walker density rises up to 0.5, there are few congested nodes on the network. But once the walkers begin to walk along the shortest path on such a network, the congestion begins to occur at about $f_c = 0.2$, as shown in Figure 1. This implies that the congestion on networks is not triggered by the initial distribution of walkers, but takes place during the process of multiple walkers walking along the shortest path between nodes. Figures 2b and 2c describe the dynamic process of congestion by plotting the number of congested nodes $N_{cong}$ (Fig. 2b) and the number of waiting walkers $n_{waiting}$ (Fig. 2c) against the number of time steps respectively. It is indicated in Figure 2 that as long as the worker density is below the critical value (for a BA scale-free network the critical value is about 0.2), few congested nodes can be found and the walkers can flow freely on the network. When the walker density reaches and goes beyond the critical value $f_c$, the walkers which initially walk freely are quickly frozen in nodes by the

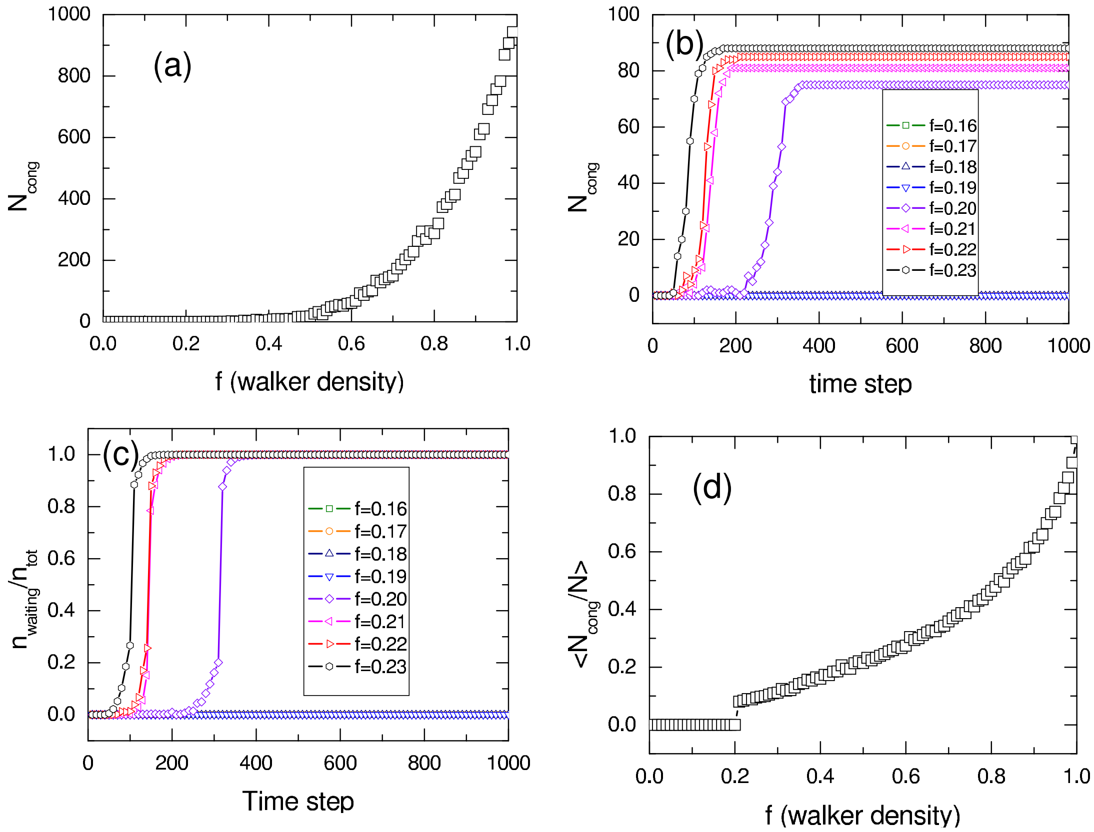


**Fig. 2.** (Color online) (a) Number of congested nodes caused by an initial distribution of walkers, (b) number of congested nodes versus time step and (c) normalized number of waiting walkers versus time step with walker densities ranging from 0.16 to 0.23 for a BA scale-free network, where $n_{tot}$ is the total number of the walkers in the network, (d) using $\langle N_{cong}/N \rangle$ as the order parameter instead of MWT, also leads to a transition at the same point. The number of time step is 10 000.

sharply increasing number of congested nodes. Although the number of congested nodes covers a small percentage at the critical point, it can still lead to a deadlock of the whole system, as shown by a transition from the free-flow phase to the congestion phase in Figure 2d.

## 4 Dependence of congestion on network topology

The influence of topology on the network congestion was also examined using a regular network, small-world network and random network constructed by WS algorithm [5] with a rewired possibility set as 0, 0.01 and 1 respectively. It is found, as shown in Figure 3, that the three kinds of networks all have their corresponding critical value of walker density $f_c$, at which the congestion transition takes place in a similar way as on the scale-free networks we discussed above. Figure 3 also shows that the critical walker density of a random network is close to that of a scale-free network, while on regular networks $f_c$ is far above that on the other networks. This phenomenon may be due to the fact that the regular network has a uniform distribution of degree $k$ and its average length of the shortest path is much longer than that of the other kinds of

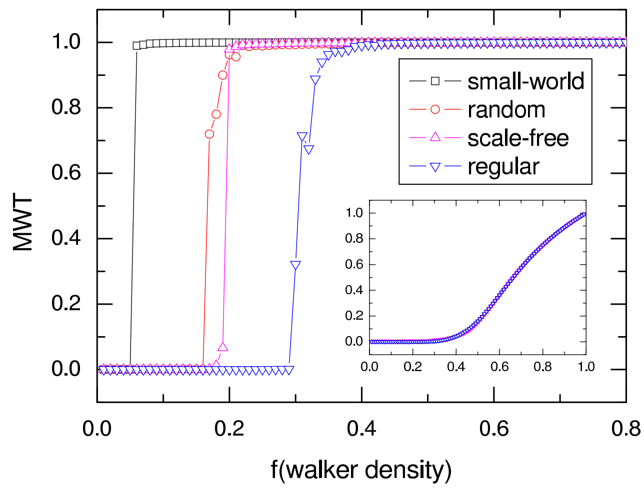


**Fig. 3.** (Color online) Mean waiting time versus walker density on the four networks with different topologies using shortest-path and random walk (inset) strategies. The number of time steps is 10 000.

networks. These make walkers distribute on regular networks more dispersively and more homogeneously than that of the other kinds of networks, and no bottle neck effect will be involved, leading to a network more tolerant to

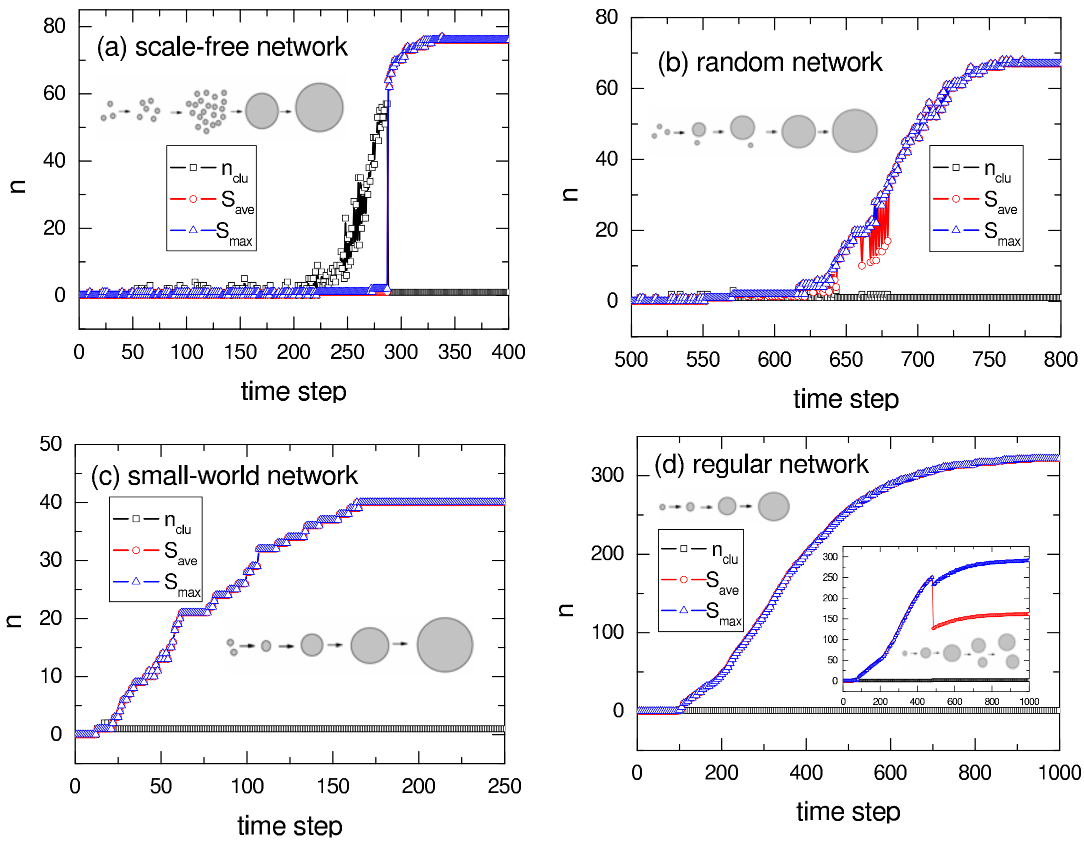


**Fig. 4.** (Color online) Number of congestion clusters, the average size and the largest size of congestion cluster versus time step for complex networks with different topologies ($N = 1000$, $\langle k \rangle = 6$, and $C_{tot} = 3\sum_i k_i$). Schematic diagrams of how congested nodes develop into connected giant congestion clusters are also shown. (a) Scale-free network (walker density $f = 0.20$). (b) Random network ($f = 0.17$). (c) Small-world network ($f = 0.06$) (d) Regular network ($f = 0.34$).

congestion. It is also found that a small-world network is much more prone to induce congestion. The reason is that a small-world network can be viewed as a homogeneous network, in which all nodes have approximately the same number of links. On one hand each node is assigned almost the same capacity according to our capacity distribution strategy, but on the other hand, because of the rewired mechanics of WS model, a little fraction of the nodes on the small world network act as the betweenness center which most of the shortest path must pass. As the bottle-neck of the network, these nodes are prevalently occupied with walkers, making the network more prone to congestion. The results obtained here are consistent with the comparison between random and scale-free networks [15] but in more general it is the regular network that is the most resistant to jamming. For a comparison between different walking strategies, we also performed a random walk on complex networks with different topologies, which is shown in the inset of Figure 3. It is found that the properties of the networks do not depend on topologies and no phase transition occurs by random walk strategy, indicating that a random walk strategy is more tolerant to network congestion, but it would take a walker much more time to reach its destination compared with the shortest-path approach [3,14].

## 5 Dynamic pictures of congestion

It is interesting to investigate the dynamics of congestion transition by examining the clusters of connected congested nodes. The evolution of congestion clusters on the four kinds of networks are studied in the vicinity of the critical walker density, as shown in Figure 4, where $n_{clu}$ means the number of congestion cluster on the network, $S_{ave}$ is defined as the average size of clusters, and $S_{max}$ is the size of the largest cluster. Before the network congestion occurs on a scale-free network, the number of congestion clusters increases with time, whereas the size of these congestion clusters remains very small and invariant. When the number of congestion clusters increases up to the critical value, all these small clusters experience a percolation transition and form a giant connected congestion cluster, which continues to grow as its neighboring nodes become congested and join the congested subnetwork. The congestion on the other three networks (Figs. 4b, 4c and 4d) starts from a small congestion cluster acting as the only nucleus, which becomes larger and larger as more and more congested nodes are connected to it once they are created, until a stable giant congested subnetwork is formed. This leads to a steady increase of both $S_{ave}$ and $S_{max}$ while keeping $n_{clu}$ at a very low level. On regular networks, sometimes splitting of a congested subnetwork into two can be found (inset of Fig. 4d) at the critical point. When the walker density is increased above $f_c$, the diversity of dynamical features among random, small-world and regular networks, such as congested subnetworks of different size isolated from each other developing from different nuclei, mergence of two clusters of different size, and the final size distribution of the isolated congested subnetworks will appear to accompany the process of nucleation [16], but the percolation/nucleation classification of dynamic properties for scale-free/non-scale-free networks remain robust against the variation of walker density above $f_c$. The different nature of congestion transition between scale-free networks and other networks lies in their different degree distribution. Scale-free networks possess a highly heterogeneous degree distribution, in which hubs, or the nodes with large degree, play a role of reservoir against congestion [17]. During the process of multiple walkers walking along the shortest path between nodes, those nodes with small degree, or with small capacity, are congested first and form into many isolated small congested clusters. Only when most of neighboring nodes of a hub are congested can the hub fail, which leads to a giant percolated congested cluster, and eventually the breakdown of the network. This phenomenon is also reminiscent of the other cascade models such as Watts model [18], where the decisions of interacting agents (nodes) are determined by the actions of their neighbors according to a simple threshold rule, or the Motter-Lai model [11,19], where the congested nodes are removed permanently. In these models, the failure of a single important node on a scale-free network may trigger a global cascade. But it rarely happens for nodes with small degree. Unlike scale-free networks, other three kinds of networks process a relatively homogeneous degree distribution free from hubs with buffer mechanism. So any single congested node may act as a nuclei, triggering an avalanche of congestion and as a result breaking down the whole network. The different dynamics of congestion for different networks implies that the network topology should be taken into consideration in the design of strategies to avoid or alleviate network congestion.

## 6 Conclusion

In summary we have investigated the dynamics of congestion transition triggered by multiple walkers walking along the shortest path on complex networks. The congestion transition on scale-free networks is a percolation process of congestion clusters, while the dynamics of congestion transition on random, small-world and regular networks is mainly a process of nucleation.

It is also found that increasing the network capacity for transporting a large number of physical quantities along the shortest path is not in every aspect an efficient strategy. One should set aside sufficient redundancy of capacity in the design, otherwise the network would be prone to congestion. To improve the effective utility of network capacity, adding more links to a network seems to be a good approach, but this is always expensive in real-life networks and only applicable to sparse networks, and thus it is not practical. The effective utility of the networks capacity can also be improved through designing a more efficient walk strategy. Random walk looks more tolerant to network congestion than shortest-path walk strategy, but more time has to be spent [9].

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